The Monty Hall Problem: A Statistical Illusion

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https://statisticsbyjim.com/fun/monty-hall-problem

Who would’ve thought that an old TV game show could inspire a statistical problem that has tripped up mathematicians and statisticians with Ph.Ds? The Monty Hall problem has confused people for decades. In the game show, Let’s Make a Deal, Monty Hall asks you to guess which closed door a prize is behind. The answer is so puzzling that people often refuse to accept it! The problem occurs because our statistical assumptions are incorrect.

The Monty Hall problem’s baffling solution reminds me of optical illusions where you find it hard to disbelieve your eyes. For the Monty Hall problem, it’s hard to disbelieve your common sense solution even though it is incorrect!

I consider the Monty Hall problem to be a statistical illusion. This statistical illusion occurs because your brain’s process for evaluating probabilities in the Monty Hall problem is based on a false intuition. Similar to optical illusions, the intuition can seem more real than the actual answer.

How to Solve the Monty Hall problem

When Marilyn vos Savant was asked this question in her Parade magazine column, she gave the correct answer that you should switch doors to have a 66% chance of winning. Her answer was so unbelievable that she received thousands of incredulous letters from readers, many with Ph.D.s! Paul Erdős, a noted mathematician, was swayed only after observing a computer simulation.

It’ll probably be hard for me to illustrate the truth of this solution, right? That turns out to be the easy part. I can show you in the short table below. You just need to be able to count to 6!

It turns out that there are only nine different combinations of choices and outcomes. Therefore, I can just show them all to you and we calculate the percentage for each outcome.

You Pick	Prize Door	STAY	SWITCH
1	1	Win	Lose
1	2	Lose	Win
1	3	Lose	Win
2	1	Lose	Win
2	2	Win	Lose
2	3	Lose	Win
3	1	Lose	Win
3	2	Lose	Win
3	3	Win	Lose
		3 Wins 0.333 (33%)	**6 Wins 0.667 (66%)**

Here's how you read the table of outcomes for the Monty Hall problem. Each row shows a different combination of initial door choice, where the prize is located, and the outcomes for when you STAY and SWITCH. Keep in mind that if your initial choice is incorrect, Monty will open the remaining door that does not have the prize.

The first row shows the scenario where you pick Door 1 initially and the prize is behind Door 1. Because neither closed door has the prize, Monty is free to open either, and the result is the same. For this scenario, if you SWITCH you lose; or, if you STAY with your original choice, you win.

For the second row, you pick Door 1 and the prize is behind Door 2. Monty can only open Door 3 because otherwise he reveals the prize behind Door 2. If you switch from Door 1 to Door 2, you win. If you STAY with Door 1, you lose.

The table shows all of the potential situations. We just need to count up the number of wins for each door strategy. The final row shows the total wins and it confirms that you win twice as often when you take up Monty on his offer to switch doors.

Why the Monty Hall Solution Hurts Your Brain

I hope this empirical demonstration convinces you that the probability of winning doubles when you switch doors. The tough part is to understand *why* this happens!

To understand the solution, you first need to understand why your brain is screaming the incorrect solution that it is 50/50. Our brains are using incorrect statistical intuition for this problem and that's why we can't trust our answer.

Typically, we think of probabilities for independent, random events. Flipping a coin is a good example. The probability of a heads is 0.500, and we obtain that simply by dividing the specific outcome by the total number of outcomes. That's why it *feels* so right that the final two doors each have a probability of 0.500.

However, for this method to produce the correct answer, the process you are studying must be random and have probabilities that do not change. Unfortunately, the Monty Hall problem does not satisfy either requirement. Its probabilities are dependent, not independent!